

Parent Handbook for Mathematics



California Department of Education
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A Message from State Superintendent of Public Instruction Jack O'Connell

We live in a world of mathematics. Every day millions of people use numbers as an integral part of their lives. We plan household budgets, figure the mileage on our cars, calculate sales tax, or measure the space in a living room. We may tabulate the receipts from a local fund-raiser or calculate the cost of food for the office picnic. Using a higher level of mathematics, we design highways and bridges, build stadiums, plan the acoustics for concert halls, and explore outer space.

Perhaps no other subject in school better teaches students the processes of logic and abstract thinking. Life requires us to contend with unknown quantities and variables and to “square” certain facts that, at first appearance, might seem to be contradictory. Mathematics trains students to be analytic and provides them with a foundation for intelligent and precise thinking.

Sometimes referred to as “the queen of sciences,” mathematics spans almost every facet of life, encompassing everything from the most mundane, everyday tasks to the wonder and order of the universe. It makes understandable what otherwise would be imponderable. It makes possible the accomplishment of routine activities and the expansion of scientific knowledge. We cannot conceive of a world in which mathematics is not fundamental and vital.

For these reasons, California’s mathematics curriculum is explicit and rigorous. It is based upon the premise that all students are capable of learning mathematics and learning it well. With persistent effort, effective instruction, and support from home, students not only will develop fluency in computational skills but will be able to understand mathematical concepts, solve problems, find solutions, reason mathematically, and communicate with precision. They will be well prepared for the demands of everyday life, the workplace, and participation as knowledgeable citizens.

In this parent handbook you will learn about the mathematics curriculum for California public schools. You will learn about the state standards at each grade level and find information on statewide testing and graduation requirements. Perhaps most important for parents, you will gain ideas for ways to help your child to be successful in this subject, for success is best achieved when schools and families work together.

I hope you and your family find this guide to be a valuable resource.



JACK O'CONNELL

Acknowledgments

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I. Introduction: The Importance of Mathematics

Mathematics has long been recognized as one of the most essential subjects in education. The reason for this recognition becomes obvious when we think of how often mathematics is used and how many situations there are in which mathematics means the difference between success or failure, life or death. For example, a wedding cake will be made large enough to feed a hundred guests only if the baker's calculations have been correct; a family will be able to make payments on a refrigerator or buy shoes and clothes for the children only by budgeting the family's income; and the safety of an aircraft depends on the mathematical calculations of aeronautical engineers. All human endeavors—agriculture, industry, trades, technology, law, sciences, crafts, and arts, for example—use in various ways this amazing branch of study.

All students need to develop proficiency in mathematics. They must learn how to think logically and solve meaningful, challenging problems. They must understand the depth and breadth of mathematical concepts and perform simple, complex, and abstract calculations accurately.

Even if students do not pursue careers that demand the study of advanced mathematics, they will find that knowledge of mathematics is essential for success in many occupations and professions. With an understanding of mathematics, students will enjoy numerous career options. They must have access to high-quality mathematics programs that provide them with opportunities to choose from a full range of future career paths. Although this ideal will not be easy to achieve, to strive for less would be a disservice to California's students and to society.

A solid grasp of mathematics gives students competence and self-assurance in the practical matters of everyday life: balancing a checkbook, purchasing a car, managing a food budget, serving as treasurer for a club or other organization, or understanding the daily news. Further, they will recognize the interdependence of mathematics and music, art, architecture, science, philosophy, and other disciplines and will become lifelong students of mathematics regardless of the careers they pursue.

Proficiency in most of mathematics is not an inborn characteristic; it is a consequence of persistence, effort, practice, support, encouragement, and effective instruction. Today, the challenges for parents and educators are to expect more from students in mathematics, raise the bar for student achievement through a more rigorous curriculum and instruction, and provide the support necessary for all students to meet or exceed those expectations. The mastery of

a challenging subject area allows students to gain confidence and satisfaction in learning.

In this handbook parents will find numerous suggestions for home or family activities that can help students succeed in mathematics. They will also find an overview of California's mathematics curriculum and requirements in mathematics for high school graduation requirements and college/university admission. The final section of the handbook offers a sample of resources that support students' mathematics education at home and in school.

II. Ways to Help Your Child Succeed in Mathematics

Educational research indicates that families are an essential part of the learning process. By doing mathematics with your children and supporting mathematics learning at home, you contribute greatly to your children's success.

There are many ways in which parents can make mathematics part of family life.

The suggestions and activities offered here include the contributions of parents and teachers, many of which were provided through the Sonoma County Office of Education and the North Bay Math Project. As you establish your own practices in supporting what your children are learning, you may consider the following ideas:

A. Always talk about mathematics in positive ways.

Regardless of your own background in mathematics be positive when you discuss mathematics with your children. Your encouragement will help ensure that they do not develop mental blocks about the subject. Work with your children to overcome trouble spots. Let them know how important mathematics is by pointing out how people use mathematics in everyday life.

B. Have high expectations for your children.

Be confident that your children can learn mathematics and then actively support them as they do so. Seek out mathematics-focused programs and activities for your children. As they get older, encourage them in their study of algebra and other courses. Encourage students to take as many advanced courses in high school as possible so that they will be prepared for whatever postsecondary option they choose.

C. Give young children a good start in mathematics.

The seeds of many important mathematics concepts are planted when children are very young, and early experiences can determine how your child looks at mathematics for the rest of his or her life. It's never too early to start learning mathematics.

Between the ages of two and four, children generally experience mathematics through simple counting. Counting is a basic and very important concept that helps children bring order to the world around them. Early counting and “how many” experiences (for example, “How many fingers am I holding up? How many crackers do you have?”) introduce children to mathematics concepts that become deeper and more complex when they are in elementary school. For example, counting three dimes later becomes a way of understanding 30 cents. The more opportunities young children have to count, the better will they understand the meaning and use of numbers—and the more confidence they will have with mathematics later on.

Parents of young children may find the following tips helpful:

- Encourage children to count frequently. Find things to count everyday, everywhere, and in every way. Start slowly with just a few things. As your children’s ability to count grows, find bigger and bigger collections of different things for them to count.
- Encourage children to count real objects: cookies, coins, toys, and so forth. Children discover that counting is more than a singsong repetition when they count real objects. Encourage children to say one number as they touch each object. Arrange objects in different ways for counting—for example, in piles, rows, and circles. Have the children count blocks as they build a tower.
- Reinforce your children’s counting. When your children finish counting, you can say, “One, two, three cookies. You counted three cookies. You have three cookies!” To correct a mistake, gently count again along with your children. Make counting a game you and your children enjoy—and play often.
- Don’t worry if your child uses his or her fingers. Fingers are the best mathematical tools children have prior to learning to write numbers—and they’re always handy and ready to use!

When your children become proficient at counting, teach them how to count by twos, fives, and even tens. Remember that counting is fundamental— a great start for learning mathematics.

D. Make mathematics a part of your family’s everyday activities.

Spend time with your children on simple board games, puzzles, and activities centered on mathematics. Involve them in activities like shopping, cooking, or home fix-it projects to show them that mathematics is practical and useful. By

pointing out the mathematics in everyday life, you can help your children learn some basic concepts and understand why mathematics is so important. Provide assistance when necessary, but let the children figure things out by themselves. Find ways to make mathematics fun!

Here are a few ideas for helping children discover—and use—the mathematics around them.

In a play area, young children can:

- Sort toys by size, kind, or color.
- Put dolls, cars, or blocks in order from largest to smallest.
- Play What Am I Thinking Of? by describing a toy's size and shape.
- Play make-believe store with toys and favorite objects.

In the kitchen, young children can:

- Look for familiar two-dimensional shapes—circles, squares, triangles, and so forth—like a round pot or a square napkin.
- Put cans of food in order by size or type.
- Sort silverware from the dishwasher to the drawer.
- Count plates, utensils, cups, or even olives.
- Divide cookies evenly so that every family member gets an equal share.
- Find how many glasses of milk are in a full milk carton.
- Help double a recipe or cut one in half.

Around the house, children of various ages can:

- Count the days on a calendar until a special event is to occur.
- Find the length and width of a room by pacing it off.
- Draw a diagram of how to rearrange furniture in a room.
- Make a map of the whole house. Older students may draw their maps to scale.

- Create a family TV schedule and track the time devoted to watching TV.
- Calculate or track the statistics of favorite athletes.
- Help an adult by calculating the number of yards of cloth needed to make curtains or clothes.

When out-of-doors young children can:

- Set up and operate a lemonade stand under adult supervision.
- Plant a garden with rows and columns of seeds.
- Count the petals on different flowers.
- Measure a sunflower or bean plant daily, keeping track of how it grows.
- Count how many times he or she can jump rope or shoot baskets in a row.
- Keep a daily chart of temperatures.
- Find triangles, squares, circles, and rectangles around the house or in the neighborhood.

When traveling, older children can:

- Use the scale on a map to calculate the approximate number of miles to a destination.
- Total the daily costs of a trip (for food, mileage, bus or train tickets, for example), and then determine the average daily expenses after the trip is completed.

E. Take opportunities to play math games with your child.

Many games provide enjoyable ways to reinforce the mathematics that children learn at school. Board games involving counting or the exchange of play money; card games requiring the tallying of points or the collection of cards in suits, kinds, or sequences; and models that require measurement and computation in order to assemble them— these are just a few of the many possible examples. Games provide not only pleasant ways for adults and children to use math together but also opportunities for older children to guide and coach younger children in applying mathematics.

F. Notice mathematics in the world at large.

Mathematics is everywhere. It is in the world that surrounds us: in nature, in the city, in the country, and even in outer space. You can help your children see the usefulness of mathematics if you point it out to them wherever you see it.

Tell your children about the mathematics you do in your job and its importance to your work. When you are outside your home, look for ways to point out mathematics: What shape does that tree look like? What is the shape of a baseball playing field, and what is the distance between bases? By counting the number of musicians in the first row of a marching band and then counting the number of rows, children can determine the total number of players in the band. Mathematics is all around us. If you start looking for examples, you will see more and more mathematics everywhere. And so will your children.

G. Become familiar with what your children are studying in mathematics.

Ask the children what they are studying in mathematics class, talk with them about mathematics homework, and help with school projects when it is appropriate. Talk with your children's teachers; feel free to ask them about the mathematics standards for your children's grade levels.

H. Help your children to use technology appropriately in mathematics.

Within the past 20 years, new technology has become a central part of everyday life. Many people in widely different occupations and vocations are finding that computers, calculators, and the Internet can be useful and valuable tools in their work.

While children need to know how to use technology in their education, it is vital that they first understand mathematics and develop the ability to use mathematics effectively *without* technology. Children must have the chance to practice basic computation and mathematical processes over an extended period of time.

When students take the state STAR tests in mathematics, they are not allowed to use a calculator. And the State Board of Education-adopted mathematics standards for California public schools do not mention technology until grade six. (Information about state standards and the state STAR tests may be found in Sections III and IV of this handbook.)

Once students are ready to use calculators and computers, however, technology provides a practical tool not only for solving certain kinds of problems but also for broadening students' opportunities and interests in mathematics. For example, students solving tasks involving long and complex calculations, charts, tables, graphs, and spreadsheets learn the power of using mathematics and technology together. If computers are not available at home, libraries and some classrooms make computers available for student use.

Helpful information regarding the use of technology in the mathematics curriculum may be found in Chapter 9 of the *Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve*, which may be accessed online at <http://www.cde.ca.gov/re/pn/fd/documents/mathematics-frame.pdf>.

In addition, the booklet *Math at Home: Helping Your Children Learn and Enjoy Mathematics*, published by the Sonoma County Office of Education, contains useful suggestions for the use of technology by students. Parents may refer to page 9 of the booklet, which may be accessed online at http://www.scoe.org/superintendent/docs/math_english.pdf.

I. Help your children at home with their mathematics homework.

As a parent, you will probably want to help your children with homework— but you may not feel prepared to help with mathematics, especially as children get older and the mathematics becomes more complex. If so, you may be pleased to learn that you do not have to be an expert to help with your children's homework!

Providing a well-lighted table and comfortable chair is an important place to start. The best location for homework is not necessarily your son's or daughter's bedroom. If your child studies at the kitchen or dining room table instead, you can help without having to sit down the entire time. You can assist when help is needed and still go about your own tasks. At the same time, you have the opportunity to keep homework time focused by giving your child support, encouragement, and gentle reminders.

A good strategy is to pass by your child's work area and periodically "check in." A quick glance will often tell you if it's time to stop and provide some extra support. When it is clear that your child could use some assistance, take a deep breath and relax. If you stay calm, it will help your child stay calm.

Many parents worry about not knowing the mathematics their children are studying. In this case, the way to provide homework help is actually quite simple:

ask questions and practice careful listening. Simple generic questions can help your child gradually make sense of mathematics, build confidence, and encourage mathematical thinking and communication. When given the opportunity to talk about mathematics, children are often able to remember what they learned in class and see the solution themselves. A good question can open up a child's thinking about the problem at hand.

Remember that listening to your child's answers and providing calm responses are as important as the questions you pose. When a child is unsure as to how to begin a problem, ask:

- Can you tell me what you know now? What facts do you have?
- What do you need to find out? Can you estimate the answer?
- Can you show me an example of this kind of problem in your book?
- How might you begin? What can you try first?
- Can you make a drawing or picture to get started?

While your child is working on a problem, ask:

- How can you organize your information? Will a list or table help?
- What would happen if ...? Show me what you did that didn't work.
- Can you explain the strategy you're using to solve this? Why did you ...?
- What could you do next? Do you see any patterns?

When your child finds an answer, ask:

- How did you get your answer? Do you think it's right? Tell me more.
- Explain why your solution makes sense.

When questions alone just won't do, another strategy for helping your child is to identify a friend or relative who knows more mathematics than you do. Find out if that person would be willing to answer an occasional phone call from your son or daughter.

The Internet can be a helpful resource because some Web sites offer homework assistance with mathematics. Librarians or library/media teachers can assist parents and children in exploring the following:

The Math Forum's Ask Dr. Math: <http://forum.swarthmore.edu/dr.math/>

Tutor.com: <http://www.tutor.com/>

BigChalk.com: <http://www.bigchalk.com/>

J. Seek assistance when your child experiences difficulty in mathematics.

If your child is having difficulty in a mathematics class, talk to the teacher, school counselor, or principal. They can recommend strategies to assist your student or provide resources that your child may find helpful. Parents might also want to explore after-school programs, summer school, and tutoring options, as well as print and Web resources designed to provide student assistance.

Additional suggestions for parents and children may be found in the booklet *Math at Home: Helping Your Children Learn and Enjoy Mathematics*, produced by the Sonoma County Office of Education:

http://www.scoe.org/superintendent/docs/math_english.pdf.

For samples of mathematics resources, both in print and on the Internet, see "A Sample List of Resources for Parents and Students" at the back of this handbook.

III. California's Mathematics Curriculum

A. Standards and Frameworks: What They Are and the Role They Play

State law requires the California State Board of Education to establish policies for each major subject area taught in public schools. Typically, the State Board sets forth these policies in the form of *content standards* and *curriculum frameworks*. Because these two terms figure prominently in discussions about public school curriculum— and because they are fundamental in determining what students should learn and teachers teach— a brief explanation is provided here.

Content standards are written expectations for what all students at a given grade level should know and be able to do. The expectations are high; they are comparable to the academic standards of countries that have high levels of student achievement. Content standards define the skills and knowledge that students need in order to be literate, educated citizens and to enter a college or university. Standards provide the basis for statewide tests that students must take at certain grade levels. District and school administrators, classroom teachers, universities that prepare teachers, and publishers of textbooks and other instructional materials pay close attention to the content standards in their work.

A curriculum framework describes the content of the course for each grade level, kindergarten through the grade twelve, and offers suggestions to teachers on *how* to teach the curriculum. A framework is a kind of “blueprint” for implementing the content standards adopted by the State Board of Education. Many teachers and administrators use a framework as a guide to help them coordinate what they will teach. Local school boards sometimes base their own curriculum decisions upon the frameworks adopted by the State Board. Many teacher education programs use frameworks as a source for professional learning. A framework also informs textbook publishers about the kinds of instructional materials needed in schools.

The State Board's content standards and framework for mathematics outline a rich program of studies for all the children in public schools. The standards are based on the premise that all students are capable of learning and using rigorous mathematic skills, concepts, and tools. In addition, the standards ensure that students at the same grade level learn similar concepts and skills regardless of what public school they attend in California.

An overview of the mathematics curriculum appears later in this section, but parents are welcome to read in their entirety California's standards and framework for mathematics. Both the *Mathematics Content Standards for California Public Schools, Kindergarten Through Grade Twelve*, and the *Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve (2000 Revised Edition)*, may be found online at: <http://www.cde.ca.gov/ci/ma/cf>.

In addition, hard copies of both publications are available for purchase from CDE Press. For price and ordering information, interested parents may e-mail CDE Press at sales@cde.ca.gov or call (800) 995-4099.

B. The Broad Goals for Mathematics Education

"All students are capable of understanding mathematics, given the opportunities and encouragement to do so."

—*Mathematics Content Standards for California Public Schools*,
California Department of Education, 1997

The California mathematics standards for kindergarten through grade twelve have changed the teaching and learning of mathematics. The mathematics curriculum at each grade level is designed to ensure that every student has access to high-quality mathematics programs and opportunities to build a firm foundation in mathematics.

The curriculum begins with basic mathematics and keeps growing in scope and content as the years progress. Mathematics education must provide students with a balanced instructional program in which students gradually develop understanding of concepts, proficiency in basic computational skills, and ability to solve problems.

The goal in mathematics education is for students to:

- Develop fluency in basic computational skills.
- Develop an understanding of mathematical concepts.
- Become mathematical problem solvers who can recognize and solve routine problems readily and can find ways to reach a solution or goal where no routine path is apparent.
- Communicate precisely about quantities, logical relationships, and unknown values through the use of signs, symbols, models, graphs, and mathematical terms.

- Develop logical thinking in order to gather data, analyze evidence, and build arguments to support or refute hypotheses.
- Develop logical thinking in order to analyze evidence and build arguments to support or refute hypotheses.
- Make connections among mathematical ideas and between mathematics and other disciplines.
- Apply mathematics to everyday life and to develop interest in pursuing advanced studies in mathematics and a wide array of related career choices.
- Develop an appreciation for the beauty and power of mathematics.

C. What Parents Should Expect Their Children to Know in a Standards-Based Curriculum

Kindergarten Through Grade Seven

In kindergarten through grade seven, the mathematics standards are divided into five topics or strands:

- Number Sense
- Algebra and Functions
- Measurement and Geometry
- Statistics and Probability
- Mathematical Reasoning

Each of the strands consists of many mathematics concepts. Students engage in deeper, more sophisticated learning within each strand as they advance through the grades. Each grade level set of standards includes all five strands.

The following information summarizes what students are expected to know and be able to do in mathematics for each grade level:

Kindergarten

By the end of kindergarten, students understand the consistency of small numbers, quantities, and simple shapes in their everyday environment. They count, compare, describe and sort objects, and develop a sense about properties and patterns.

Grade One

By the end of the first grade, students understand and use the concept of ones and tens in the place value number system. They add and subtract small numbers with ease, measure with simple units, and locate objects in space. They also describe data and analyze and solve simple problem situations.

Grade Two

By the end of the second grade, students understand place value and number relationships in addition and subtraction and use simple concepts of multiplication. They measure quantities with appropriate units. They classify shapes and see relationships among them by paying attention to the elements that compose them (what mathematics teachers call geometric attributes). Students also collect and analyze data and verify answers.

Grade Three

By the end of the third grade, students understand more deeply place value and show skill in performing addition, subtraction, multiplication and division of whole numbers. They also estimate, measure, and describe objects in space. They use patterns to help solve problems, represent number relationships, and conduct simple probability experiments.

Grade Four

By the end of the fourth grade, students understand large numbers and addition, subtraction, multiplication, and division of whole numbers. They also describe and compare simple fractions and decimals and understand the properties of and the relationships between plane geometric figures. Further, they collect, represent, and analyze data to answer questions.

Grade Five

By the end of the fifth grade, students increase their facility with the four basic arithmetic operations applied to fractions, decimals, and positive and negative numbers. They also know and use common measuring units to determine length and area and use formulas to determine the volume of simple geometric figures. Students know the concept of angle measurement and use a protractor and compass in solving problems and use grids, tables, graphs, and charts to record and analyze data.

Grade Six

By the end of the sixth grade, students have mastered the four basic arithmetic operations with positive and negative numbers, whole numbers, fractions, and decimals. They also compute and solve problems accurately and apply their knowledge to statistics and probability. They understand the concepts of the mean, median, and mode of data sets and can calculate the range. Further, they analyze data and sampling processes for possible bias and misleading conclusions and use addition and multiplication of fractions routinely to calculate probabilities for compound events. Students understand ratios and proportions conceptually and work with them. They also compute percentages (e.g., tax, tips, interest). They know about π and the formulas for determining the circumference and area of a circle. They use letters for numbers in formulas involving geometric shapes and in ratios to represent an unknown part of an expression. They also solve one-step linear equations.

Grade Seven

By the end of the seventh grade, students are adept at manipulating numbers and equations and understand the general principles at work. They understand and use the factoring of numerators and denominators and properties of exponents. Knowing the Pythagorean Theorem, they solve problems in which they compute the length of an unknown side. They also know how to compute the surface area and volume of basic three-dimensional objects, understand how area and volume change with a change in scale, and make conversions between different units of measurement. Knowing and using different representations of fractional numbers (fractions, decimals, and percents), They are proficient in changing from one to another. They increase their facility with ratio and proportion and compute the percentages of increase and decrease and simple compound interest. They also graph linear functions and understand the idea of slope and its relation to ratio.

Grades Eight Through Twelve

After studying the five strands of mathematics from kindergarten through grade seven, students concentrate on specialized areas of mathematics in greater depth in grades eight through twelve. *Note:* The mathematics standards for grades eight through twelve are organized by disciplines rather than by strands:

- Algebra I
- Geometry

- Algebra II
- Trigonometry
- Mathematical Analysis
- Linear Algebra
- Probability and Statistics
- Advanced Placement Probability and Statistics
- Calculus

Mathematical reasoning and conceptual understanding are included and are necessary in every mathematical discipline that students study at these more advanced levels.

The following information summarizes what students are expected to know and be able to do in the various mathematics disciplines:

Algebra I

In algebra students learn to reason symbolically. Consequently, the complexity and types of equations and problems that they are able to solve increase dramatically. The key content for the first course, Algebra I, involves understanding, writing, solving, and graphing linear and quadratic equations, including systems of two linear equations in two unknowns. Quadratic equations can be solved by factoring, completing the square graphically, or applying the quadratic formula. Students should also become comfortable with operations on monomial and polynomial expressions. They learn to solve problems by employing all of these techniques and thereby extend their mathematical reasoning in many important ways, including justifying steps in an algebraic procedure and checking algebraic arguments for validity.

Because symbolic reasoning and calculations with symbols are central to algebra, students who study algebra develop an understanding of the symbolic language of mathematics and the sciences. In addition they develop and use algebraic skills and concepts in a wide variety of problem solving situations.

Geometry

The main purpose of the geometry curriculum is to develop geometric skills and concepts and the ability to construct formal logical arguments and proofs in a geometric setting. Although the curriculum stresses plane (synthetic) Euclidean

geometry, there is room to place special emphasis on coordinated geometry and its transformations.

The geometric skills and concepts developed in this discipline are useful to all students. Aside from these skills and concepts, students develop ability to construct formal logical arguments and proofs in geometric settings and problems.

Algebra II

Algebra II expands on the mathematical content of Algebra I and geometry. It introduces many new concepts and techniques that will be basic to more advanced courses in mathematics and the sciences and useful in the workplace. Generally, abstract thinking skills, the function concept, and the algebraic solution of problems in various content areas are emphasized.

Students who master Algebra II gain experience with algebraic solutions of problems in various content areas, including systems of quadratic equations, logarithmic and exponential functions, the binomial theorem, and the complex number system.

Trigonometry

Trigonometry uses the techniques learned previously in algebra and geometry courses. Trigonometric functions are defined geometrically rather than algebraically. Facility in dealing with those functions and ability to prove basic identities regarding them are especially important for students who intend to study calculus, more advanced mathematics, physics and other sciences, and engineering in college or university.

Mathematical Analysis

Mathematical Analysis combines many of the trigonometric, geometric, and algebraic techniques needed in preparing for the study of calculus and helps students strengthen their conceptual understanding and mathematical reasoning when solving problems. The content standards for Mathematics Analysis provide a functional point of view toward those topics. The most significant new concept is that of limits. Mathematical analysis is often combined with trigonometry or perhaps linear algebra to make a yearlong precalculus course.

Linear Algebra

The general goal of linear algebra is that students learn the techniques of matrix manipulation so that they can solve systems of linear equations in any number of variables. Linear algebra is most often combined with another subject, such as trigonometry, mathematical analysis, or precalculus.

Probability and Statistics

Probability and statistics introduces the study of probability, interpretation of data, and fundamental statistical problem solving. Mastery of this academic content provides students with a solid foundation in probability and facility in processing statistical information.

Probability and Statistics, Advanced

Advanced probability and statistics is a technical, in-depth extension of probability and statistics. In particular, mastery of advanced placement academic content gives students the background for success in the Advanced Placement examination in the subject. (For information regarding advanced placement, see the following section, titled “Planning for Your Child’s Success in Mathematics: Requirements and Course Decisions.”)

Calculus

When taught in high school, calculus should be presented with the same depth and rigor as provided in entry-level college and university calculus courses. The content standards for calculus outline a complete college curriculum in one variable calculus. Admittedly, many high school programs may not have enough time to cover all of the following content in a typical academic year. For example, some school districts may treat differential equations lightly and spend a substantial amount of time on infinite sequences and series; others may do the opposite. Consideration of the College Board syllabi for the Calculus AB and Calculus BC advanced placement sections of the *Advanced Placement Examination in Mathematics* may help districts make curricular decisions. Calculus is a widely applied area of mathematics, involving a beautiful intrinsic theory. Students mastering this content will be exposed to both these important aspects.

D. District Options for Grades Eight Through Twelve

To allow school districts, schools, and teachers flexibility in teaching these high school disciplines, the standards for grades eight through twelve do not mandate that a particular course be initiated or completed in any particular grade.

Moreover, the subject matter for these various disciplines is sometimes combined differently. For example, some schools teach the standards in traditional mathematics courses, such as trigonometry and Algebra II. Other schools teach the standards in integrated or blended fashion. That is, one school might combine some trigonometry, mathematical analysis, and linear algebra to form a precalculus course; another school might offer trigonometry content together with Algebra II.

IV. Planning for Your Child's Success in Mathematics: Requirements and Course Decisions

A. STAR— The Standardized Testing and Reporting Program

California law mandates statewide testing of certain subjects taught at specific grade levels. The purpose is to determine student achievement by school district, by school, and by the individual student. Called the Standardized Testing and Reporting (STAR) program, the testing program consists of three basic components: the *California Standards Tests (CSTs)*, the *California Achievement Test, Sixth Edition Survey (CAT/6)*, and the *California Alternate Performance Assessment (CAPA)*.

California Standards Tests

All students in grades two through seven are required to take the *CST* for mathematics. From grades eight to eleven, students take the *CST* corresponding to the courses in which they are enrolled. For example, students enrolled in Algebra I take the *CST* in Algebra I, students enrolled in Geometry take the *CST* for Geometry, and so on. In addition to the Algebra I and Geometry *CSTs*, there are *CSTs* in Algebra II, general mathematics, high school summative mathematics, and integrated mathematics. Students in grades eight and nine not enrolled in a specific mathematics course take the general mathematics *CST*. Students who have completed Algebra II or Integrated Mathematics III take the high school summative mathematics test.

The *CAT/6*

The *CAT/6 Survey* is a nationally norm-referenced test. This means that the results of the test permit a comparison of achievement with other states. The *CAT/6* is given to students in grades two through eleven. Among the knowledge and skills tested by the *CAT/6* are the following:

Number and Number Relations

Problem Solving and Reasoning

Operation Concepts

Computation and Numerical Estimation

Measurement

Patterns, Functions, and Algebra

Geometry and Spatial Sense

Data Analysis, Statistics, and Probability

The *CAPA* is designed for students with significant cognitive disabilities who are unable to take the *CSTs* or the *CAT/6*. Participation in the *CAPA* is specified in the student's individualized education program (IEP). Included in the *CAPA* are mathematics tasks that measure the students' achievement of the mathematics standards selected for these students.

Teachers, parents, and students can use individual test results to help monitor a student's progress. Additional information about STAR may be found on the California Department of Education's Web site at <http://www.cde.ca.gov/ta/tg/sr>.

B. High School Graduation Requirements and Options

Courses that incorporate California's mathematics standards are rigorous and demanding. They are designed to prepare students for the challenges they will face in a career and in higher education, both of which routinely require a comprehensive knowledge of mathematics.

You can help students become successful in mathematics by understanding what they are required to learn and by stressing the importance of hard work. Be sure that they know that you have confidence in their ability to succeed in mathematics. Continue to speak positively about mathematics and support their learning actively.

If your child is experiencing difficulty in a mathematics course, discuss the problems with the teacher, school counselor, or principal. They can recommend strategies to assist your student or provide helpful resources. Parents might also want to explore tutoring options, after-school programs, summer school, or print and Web resources designed to provide student assistance.

Two years of mathematics are required for graduation from a California high school, including Algebra I. If a student completes Algebra I in grade seven or grade eight, he or she must also take two additional years of mathematics in order to meet the state's graduation requirements. Local districts may add to the state minimum requirements, so parents and students should confer with local teachers and counselors.

Algebra I Requirement

In 2003-04 state law began requiring that students satisfactorily complete a course or combination of courses based upon state content standards for

Algebra I in order to graduate from high school. Because algebra is considered the foundation of all higher-level mathematics and essential for many career choices, students must master it to advance to other classes and graduate from high school. To ensure that students are successful in high school algebra, schools are increasingly emphasizing instruction in the beginning concepts of algebra throughout the elementary school and middle school years.

Does every student need algebra? Many researchers and teachers indicate that all students, regardless of their prior mathematical skills, benefit from taking algebra. Therefore, every student should be prepared for and be provided with opportunities to study algebra under high-quality teachers with access to the resources and support systems that will promote student success. Algebra is the passport to advanced mathematics, success in college and university, and success in tomorrow's careers. Without a fundamental understanding of algebra, students may be unable to succeed in completing the courses needed for admission to institutes of higher education.

Algebra builds on arithmetic to develop more sophisticated tools to complete tasks, such as determining which is the most cost-effective telephone plan or how much to invest at a particular interest rate to earn a certain amount. Algebra can also instill in students logical thinking and persistence. The U. S. Department of Labor reports that the highest paying, fastest-growing job sectors all require significant technical skills. Algebra is the language of mathematics, which itself is the language of the information age. Every effort must be made to keep the doors of opportunity open for everyone to study algebra.

Advanced Placement Courses

If students are highly successful in mathematics, they may be offered opportunities to take advanced placement (AP) mathematics courses in high school. Offering students the highest level of mathematics study available in high school, the courses can earn college credits. Taking AP mathematics courses provides a distinct advantage for college-bound students because applicants who have excelled in completing these courses are given special consideration by colleges and universities. Teachers of mathematics and counselors can provide information to students or parents regarding AP courses and eligibility.

California High School Exit Exam

Beginning with the graduating class of 2006, California high school students will be required to take the *California High School Exit Examination (CAHSEE)* in grade ten. Students who do not pass the test the first time they take it will have opportunities to take it again in grades eleven and twelve. The purpose of the examination is to improve student achievement in high school and help ensure

that students who graduate from high school demonstrate competency in state content standards.

CAHSEE has two parts, English–language arts and mathematics. The mathematics portion is designed to test students' grasp of the mathematics topics outlined in the grade seven standards through the first half of the Algebra I standards. It includes statistics, data analysis and probability, number sense, measurement and geometry, mathematical reasoning, and algebra. Students must demonstrate strong computational skills and a foundation in arithmetic, including an ability to work with decimals, fractions, and percentages. The mathematics portion of the exam consists entirely of multiple-choice questions.

More information about the *California High School Exit Examination*, including sample problems, may be available at your local high school. The California Department of Education offers additional information on the Internet at: <http://www.cde.ca.gov/ta/tg/hs/>.

C. Mathematics Requirements for College and University Entrance

A minimum of three years of approved college preparatory mathematics courses is required for admission as a freshman into the University of California (UC) or the California State University (CSU) system. (Four years of mathematics is recommended, especially for the UC.) The three approved courses must include at least Algebra I, geometry, and intermediate algebra.

The mathematics subject requirement for freshman admission to the University of California includes the subject requirement, the scholarship requirement, and the examination requirement. The students must meet the scholarship requirement by attaining a minimum GPA in required subjects. All freshman applicants must also submit their scores on the *SAT 1* or the *American College Test*, as well as three *SAT II: Subject Tests*, including *Mathematics Level 1c* or *2c*.

In considering college and university requirements, parents and students begin planning while the student is in middle school and should discuss admission

requirements with high school counselors. They should also refer to the following Web site for information on admissions to UC and CSU: www.californiacolleges.edu.

Courses required by UC are sometimes referred to as the a–g requirements because of the order in which the requirements are listed. Students and parents can view by high school the university-approved courses that help to meet this requirement: www.ucop.edu/doorways/list.

V. A Sample List of Resources for Parents and Students

Whether in school or at home, students can enjoy and understand mathematics concepts more readily by reading good books. By combining mathematics and children's literature, parents can offer young students a learning experience that touches both worlds. Students who are gaining proficiency at counting, for example, enjoy such books as Wanda Gag's *Millions of Cats* and Mitsumasa Anno's *Anno's Counting Book*. Later, mathematics-related literature can help students understand the relationship of mathematics to other school subjects. For example, *The Doorbell Rang* by Pat Hutchins offers an example of scarcity, an economics concept commonly explored in history–social science during the primary grades. And David Bodanis's *E=mc²: A Biography of the World's Most Famous Equation* helps high school students learn how mathematics relates to world history and energy, mass, and light.

Parents are encouraged to accompany their children on trips to public or school libraries. There librarians or library/media teachers can help locate literature relating to mathematics. In addition, parents and teachers are encouraged to peruse online the California Department of Education publication, *Literature for Science and Mathematics, Kindergarten Through Grade Twelve*, for a number of mathematics-related titles. This extensive bibliography is helpful in identifying age-appropriate books available in libraries and bookstores. The list may be accessed at <http://www.cde.ca.gov/ci/sc/ll/index.asp>.

Some county offices and school districts have developed lists of literature that complement the study of various subjects. Parents can consult with library/media staff in school districts or county offices to find out what resources are available in their vicinity.

Other Printed Materials

A number of publishers have produced helpful resources that range from activity books and instructional materials to official state publications.

Algebra to Go. Great Source Education Group, 1998. This small handbook is a handy reference tool for students and parents. It provides short definitions, examples, and lessons on the concepts students study from grade five through Algebra I.

Family Math by Jean Stenmark, Virginia Thompson, and Ruth Cossey. Lawrence Hall of Science, University of California, Berkeley Press, 1986.

Family Math is a popular book containing dozens of mathematics activities that parents and children ages eight to twelve can do together. Included are activities related to number sense, geometry, probability and statistics, and algebra. A Spanish version of the book, *Matemática Para La Familia*, is also available.

Family Math for Young Children, by Grace Coates and Jean Kerr Stenmark. Lawrence Hall of Science, University of California, Berkeley Press, 1997. A sequel to *Family Math*, this book was developed for families with children ages of four through eight.

Family Math—The Middle School Years, Virginia Thompson and Karen Mayfield-Ingram. Lawrence Hall of Science, University of California, Berkeley Press, 1998. The activities suggested in this book cover algebraic reasoning and number sense and are appropriate for students in grades six through eight.

Helping Your Child Learn Math, by Patsy Kanter and Linda Darby. U.S. Department of Education, 1999. This booklet offers advice for parents, suggested resources, and mathematics activities for children in kindergarten through grade six. It is available online at <http://www.ed.gov/pubs/parents/Math/>.

Helping Your Child with Homework. California Department of Education, 1995. This publication answers questions that parents of children in elementary and junior high school often ask about homework. Some ideas may also be helpful for high school students. For price and ordering information, contact the publisher at sales@cde.ca.gov or call (800) 995-4099.

Math at Home: Helping Your Children Learn and Enjoy Mathematics. Sonoma County Office of Education, 2002. This public service publication has served as a major source of the information included in this handbook. In addition to suggesting activities for home and study, the booklet lists nearly 40 selections of children's literature that support mathematics in kindergarten through grade four. Copies of the booklet can be obtained through the Sonoma County Office of Education, Curriculum and Instruction Division. Telephone (707) 524-2600, or go online at http://www.scoe.org/superintendent/docs/math_english.pdf.

Math Matters: Kids Are Counting on You. PTA Kit from the National Academy Press, 1998. PTAs and other parent groups that want to help teachers and administrators bring mathematics to the forefront of their

elementary school will find this kit helpful. A Spanish version is also available.

Math On Call. Great Source Education Group, 1998. Short definitions, examples, and lessons on over 300 mathematics concepts studied in kindergarten through grade eight are included in this small handbook for students and parents.

Mathematics-Related Web Sites

With the growth of the Internet, a number of mathematics-related sites have become available. In addition to Web sites listed elsewhere in this handbook, the following may also prove helpful to parents, students, and teachers:

Figure This! includes a collection of mathematical challenges for middle school students and their families. Each challenge comes with a hint and the complete solution, along with related information and questions to think about: www.figurethis.org.

Federal Resources for Educational Excellence includes information and resources for parents and parent groups: <http://www.ed.gov/free/>.

The Math Forum's Student Center hosts *Ask Dr. Math* and has weekly and monthly math challenges, Internet math hunts, and math resources organized by grade level: <http://forum.swarthmore.edu/>.

The National Council of Teachers of Mathematics, the national professional organization of mathematics educators, includes information on national standards in mathematics education, kindergarten through grade twelve: <http://www.nctm.org>.

Math Is Power, one of the information Web sites of the National Science Foundation, links students to Internet sites that provide homework help and Web-based mathematics resources: www.mathispower.org.

VI. For More Information from the California Department of Education

For general information regarding the content standards and frameworks or the process for the state adoption of mathematics instructional materials, please contact Thomas Adams, Director, Curriculum Frameworks and Instructional Resources Division, (916) 319-0881.

For information regarding statewide testing in mathematics, please contact the Standards and Assessment Division, (916) 445-9441.

For information regarding mathematics curriculum and instruction, including the California Department of Education's publication, *Literature for Science and Mathematics, Kindergarten Through Grade Twelve*, please contact Phil Lafontaine, Manager, Mathematics and Science Leadership Office, (916) 323-5847.

For information on family, parent, and community involvement, please contact the Title I Policy and Partnerships Office, under whose guidance this handbook was prepared, at (916) 319-0854.